DEVELOPMENT AND APPLICATIONS OF A SELF-STARTING ONE-STEP CONTINUOUS HYBRID BLOCK COLLOCATION INTEGRATOR FOR INITIAL VALUE PROBLEMS

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ABSTRACT

Many real life problems are usually resolved into mathematical models, called differential equations. Solving differential equation implies solving the problems represented by them. The paper reports the development and applications of a self-starting implicit continuous hybrid block collocation method (CHBCM) for integration of initial value problems of ordinary differential equations. Power series polynomial was adopted as an approximate solution. Using interpolation and collocation at selected hybrid points, a class of self-starting one-step hybrid block method was obtained. The analysis shows that the method was zero stable, consistent and convergent. When applied on sample standard problems, the scheme performed well as it produced exact solutions up to seven decimal places, thereby competing favourably with similar existing methods. This new method is therefore recommended since they are self-starting, requires fewer function evaluation.

Key words: *Initial value, differential equation, linear multistep, collocation, continuous, Hybrid, power series polynomial, one-step, interpolation, block, solutions, zero stability, consistency and convergence.*

1.0 INTRODCUCTION

A differential equation is a mathematical depiction of the relationship between a dependent variable, y and its derivative in a dynamical system `(Yang, Cao, Chung and Morris, 2005). Most real life problems in science, engineering and technology are often resolved into differential equations. As such, solving differential equation is a daily activity for mathematicians, engineers and other scientists. A differential equation involving ordinary derivatives is called ordinary differential equation (ODE) (Phillips and Woodford, 2012), generally represented in the form,

$$y = f(x, y); h(x_0) = y_0, a \leq x \leq b \qquad (1)$$

where,(x_0, y_0) is the initial condition.

The solution of (1) is a function $y(x)$ which satisfies the equation and the stated initial conditions. Unfortunately, very few of this type of equation have analytic or closed form solutions. Where such solutions exist, they cannot be easily used in resolving practical or real life problems. Hence the need for approximate solutions, called numerical solution, usually obtained by numerical quadrature (Ross, 2010; Kandell, Han and Stewart, 2009). Numerical solutions are highly utilized in

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practice because they are often found in a form that is readily useful and are easy to interpret and implement. Besides, their use had been simplified by the advent modern computers.

The search for numerical solutions originated from the works of *Jacob Bernoulli* who established the *convergence theorem,* a process meant to make precise the solution of differential equations as opposed to the work of his predecessors (Euler and Newton), who employed *quantitative and analytic methods* (Hairer, Norsett and Wanner, 2008). This theorem formed the basis for the *existence and uniqueness theorem* which guarantees the existence and uniqueness of the solution of a differential equation in the closed interval $[a, b]$. Numerical methods can be classified into two groups, the one step and multistep. The one step methods have been described and applied by many scholars (Lambert, 1973, 1991, Butcher, 2008 and Suli, 2014) and have been observed to be self-starting and easy to implement but they are beset by many limitations. These include: (i) requirement for much computational efforts (Suli, 2014); (ii) difficulty in introducing additional information; (iii) excessive function evaluation; (iv) time consumption and low accuracy (Anake, 2011); (v) the need for reduction of order; (vi) cumbersome computer program and (vii) the equation cannot be solved explicitly.

Multistep methods were developed to address the weaknesses of the one step methods. The multistep methods gives higher order of accuracy and are suitable for the direct solutions of higher order equations without reduction of order. The challenge of the multistep methods are that they are expensive to implement and the predictor often have lower order of accuracy (Anake, 2011). More so, they require starting values which are not always available or may contain error. Besides, they are also affected by the Dahlquist Barrier (Hairer, Norsett and Wanner, 2008). This situation led to the search for Hybrid methods.

According to Lambert (1973) a hybrid method, modified multistep method, was introduced to address the limitations of the linear multistep methods and to circumvent the Dahlquist Barrier. Gupta (1979) states that hybrid methods are so-called because they combine the features of Runge-Kutta (one step) and the linear multistep methods. The advantages of this is that it is self-starting, one step methods, permits easy change of steps, less expensive, involves less function evaluation and yet produces a simultaneous solution at all grid points (Lambert 1973). In recent times many hybrid methods have been developed and implemented. Most of them results to high order yet the result seems less accurate as compared to exact solutions (Odejide and Adeniran, 2012; Odekunle, Adesanya and Sunday, 2012 and Umaru, Aliyu, I. M and Aliyu, Y. B, 2014).

This paper propose a class of new self-starting implicit one step continuous hybrid block collocation method using two off-point on the interval $[a, b] \in \mathbb{R}$. Thus the aim of the research was to develop and apply a class of one step continuous hybrid block collocation method (CHBCM) for solving first order initial value problems of the type in equation (1) above on the interval $[a, b]$; $a, b \in \mathbb{R}$, partitioned at equal steps size $h = x_i - x_{i-1}, i = 1(1)n$ using two off-points, $v_1, v_2 \in [a, b]$. This paper reports the: (1) derivation of a continuous hybrid block collocation method for integration of general first order initial value problem; (2) analysis of the properties of the method in terms of its consistency, zero stability and convergence; (3) implementation of the method in integrating standard first order initial value problems; (4) performance of the new method by comparing the results with exact solutions and those obtained from similar existing methods.

2.0 METHODOLOGY

The method used for the development of linear multistep methods include Taylor's expansion, numerical integration or polynomial interpolation (Lambert, 1973, 1991, Miletics and Molnarka, 2010 and Suli, 2014). In this research the interpolation approach was applied. The basic assumption made was that equation (1) has a power series polynomial solution of the form $p(x) = \sum_{0}^{m} a_j x^j = y_{n+r}$; where, $a_j, j = 0(1)m$ are unknowns to be determined, $m = r + s - 1$ is the degree of the polynomial, x^j to be determined, x^j is a polynomial basis function for the approximation. The power series was interpolated at carefully selected off-points while its first derivative was collocated at all points. After appropriate substitutions and rearrangements we have a system of $m+1$ equations in $m+1$ unknowns a_j. Solving the resulting system for the coefficients a_j by matrix inversion using Maple Software and substituting them into the power series, we obtain a new continuous hybrid method in form of a linear multistep method. *Evaluating* the continuous method at the off-points and the grid-points we have $m-1$ discrete schemes combined to form a block scheme. The method was analyzed for zero-stability, consistency and convergence. The region of absolute stability of the method was plotted using the boundary locus method with the aid of Maple Software. Finally, a Maple code was developed to test the performance of the new method on sample standard initial value problems. The results obtained were compared with exact solutions and those obtained from existing methods.

3.0 DERIVATION

Let the solution of equation (1) be a power series polynomial of the form,

$$y(x)=\sum_{0}^{m} a_j x^j = y_{n+r} \tag{2}$$

where, $a_j j = 0(1)m$ are unknown parameters to be determined, x^j is a polynomial basis function of degree $m = r + s - 1$, while r and s are the number of distinct interpolation and collocation points respectively.

Differentiating equation (2) once and substituting the derivative into equation (1) we get,

$$\sum_{j=1}^{m} j a_j x^{j-1} = f_{n+s}, \quad s=0(v_i)k,\ i=0(1)s \tag{3}$$

Interpolating equation (2) at selected members of the set $x_{n+r}, r=0(v_i)k, i=1(1)r$ of interpolation points, where v_i are hybrid (off-points), we get the system,

$$\sum_{j=0}^{m} a_j x^j = y_{n+r} \tag{4}$$

Collocating equation (3) at all the points of the step under consideration, including off-step points, x_{n+s}, $s = 0(v_i)k$, $i=1(1)s$, where k is the number of collocation points, and v_i the off-step points respectively and k is the step number of the method, we have the following system of equations is,

$$\sum_{j=1}^{m} j a_j x^{j-1} = f_{n+s}, \quad s=0(v_i)k; i=1(1)s \tag{5}$$

Combining equations (4) and (5), expanding and then converting to matrix form we get,

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & \dots & x_n^m \\ 1 & x_{n+v_1} & x_{n+v_1}^2 & x_{n+v_1}^3 & \dots & x_{n+v_1}^m \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{n+v_m} & x_{n+v_m}^2 & x_{n+v_m}^3 & \dots & x_{n+v_m}^m \\ 0 & 1 & 2x_n & 3x_n^2 & \dots & Nx_n^{m-1} \\ 0 & 1 & 2x_{n+v_1} & 3x_{n+v_1}^2 & . & Nx_n^{m-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 1 & 2x_{n+v_m} & 3x_{n+v_m}^2 & \dots & Nx_n^{m-1} \\ 0 & 1 & 2x_{n+k} & 3x_{n+k}^2 & \dots & Nx_{n+k}^{m-1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \dots \\ . \\ . \\ . \\ \dots \\ . \\ a_{r+s-1} \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+v_1} \\ \dots \\ y_{n+v_m} \\ f_n \\ f_{n+v_1} \\ \dots \\ f_{n+v_m} \\ f_{n+k} \end{bmatrix} \tag{6}$$

Solving for $a_j,\ j=0(1)m$ in equation (6) by matrix inversion and substituting the values into equation (2) and rearranging, we get the continuous k-step Continuous hybrid block collocation scheme in form of continuous linear multistep method given by,

$$y(x)=\sum_{j=0}^{k}\alpha_j(x)y_{n+j}+\sum_{v_i}\alpha_{v_i}(x)y_{n+v_i}+h[\sum_{j=0}^{k}\beta_j(x)f_{n+j}+\sum_{v_i}\beta_{v_i}(x)f_{n+v_i}] \quad (7)$$

Where, $y_{n+j}=y\left(x_{n+j}\right)$ and $f_{n+j}=f(x_{n+j},y_{n+j},y'_{n+j})$. The coefficients $\alpha_j(x)$ and $\beta_j(x)$ are continuous coefficients and k is the order of the problem. Evaluating the method in equation (7) at $x=x_{n+v_i},i=1(1)m$, where v_i are off-points, we get $m-1$ discrete schemes of the form.

$$\sum_{j=0}^{k}y_{n+v_i}-y_n=h\sum_{j=0}^{k}(f_{n+j}+f_{n+v_i}),i=1,2,...,r;\ j=0,1,2...,k \quad (8)$$

This is the general form of the proposed hybrid method, which can be written in compact block vector form (James *et al*, 2013; Olabode and Omole, 2015) as

$$A^O Y_m=A^1 y_m+h^{\gamma}[BF(y_m)+CF(Y_m)] \quad (9)$$

Where,

A^0, A^1, B and C are constants rxr matric efficient matrices associated with the vectors $Y_m=(y_{n+v_i},y_{n+v_i})^T$, $y_m=(y_n,y_n)^T$, $F(Y_m)=(f_{n+v_i},f_{n+k})^T$, $F(y_m)=(f_n)$ and $\gamma=1$ is the order of the derivative and h is the step size of the method.

4.0 SPECIFICATION

To obtain the required case of one-step ($k=1$) method with two off-step points $v_i,i=1,2$, there is need for specification. To this end, we select two off-points $v_1=\frac{1}{3}$, and $v_2=\frac{2}{3}$ while the number of collocation points $s=4$, $r=1$. Assuming the solution to be a polynomial of degree $m=r+s-1=4$ of the form,

$$y(x)=\sum_{j=0}^{4}a_j x^j=y_{n+r} \quad (10)$$

Applying the derivation procedure in equations (4) to (6) above for $k=1$, $s=4$ and $r=1$, yields the following system of 5-equations in 5-unknowns; a_0, a_1, a_2, a_3 and a_4,

$$\begin{aligned}
a_0+a_1x_n+a_2x_n^2+a_3x_n^3+a_4x_n^4&=y_n\\
a_1+2a_2x_n+3a_3x_n^2+4a_4x_n^3&=f_n\\
a_1+2a_2x_{n+\frac{1}{3}}+3a_3x_{n+\frac{1}{3}}^2+4a_4x_{n+\frac{1}{3}}^3&=f_{n+\frac{1}{3}}\\
a_1+2a_2x_{n+\frac{2}{3}}+3a_3x_{n+\frac{2}{3}}^2+4a_4x_{n+\frac{2}{3}}^3&=f_{n+\frac{2}{3}}\\
a_1+2a_2x_{n+1}+3a_3x_{n+1}^2+4a_4x_{n+1}^3&=f_{n+1}
\end{aligned} \quad (11)$$

Converting equation (8) to matrix form and rearranging, we have

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$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 \\ 0 & 1 & 2x_{n+\frac{1}{3}} & 3x_{n+\frac{1}{3}}^2 & 4x_{n+\frac{1}{3}}^3 \\ 0 & 1 & 2x_{n+\frac{2}{3}} & 3x_{n+\frac{2}{3}}^2 & 4x_{n+\frac{2}{3}}^3 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 \end{bmatrix}^{-1} \begin{bmatrix} y_n \\ f_n \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{2}{3}} \\ f_{n+1} \end{bmatrix} \qquad (12)$$

Substituting $x_n = 0, x_{n+\frac{1}{3}} = \frac{1}{3}, x_{n+\frac{2}{3}} = \frac{2}{3}, x_{n+1} = 1$ into equation (9) and solving for the parameters $a_j, j = 0,1,2,3,4$ by matrix inversion with the aid of Maple Software, we have,

$$\begin{aligned} a_0 &= y_n \\ a_1 &= hf_n \\ a_2 &= -\tfrac{11}{4}hf_n + \tfrac{9}{2}hf_{n+\frac{1}{3}} - \tfrac{9}{4}hf_{n+\frac{2}{3}} + \tfrac{1}{2}hf_{n+1} \\ a_3 &= 3hf_n - \tfrac{15}{2}hf_{n+\frac{1}{3}} + 6hf_{n+\frac{2}{3}} - \tfrac{3}{2}hf_{n+1} \\ a_4 &= -\tfrac{9}{8}hf_n + \tfrac{27}{8}hf_{n+\frac{1}{3}} - \tfrac{27}{8}hf_{n+\frac{2}{3}} + \tfrac{9}{8}hf_{n+1} \end{aligned} \qquad (13)$$

Substituting the values in equations (10) into equation (7) and rearranging, produce the continuous hybrid collocation method, in the form of linear multistep method,

$$\begin{aligned} y(x) = y_n + h\{&(x - \tfrac{11}{4}x^2 + 3x^3 - \tfrac{9}{8}x^4) f_n + (\tfrac{1}{2} - \tfrac{3}{2}x^3 + \tfrac{9}{8}x^4) f_{n+1} \\ &+ (\tfrac{9}{2}x^2 - \tfrac{15}{2}x^3 + \tfrac{27}{8}x^4) f_{n+\frac{1}{3}} + (-\tfrac{9}{4}x^2 + 6x^3 - \tfrac{27}{8}x^4) f_{n+\frac{2}{3}}\} \end{aligned} \qquad (14)$$

Evaluating the continuous hybrid method in equation (14) at the points, $x = \{x_{n+1}, x_{n+\frac{2}{3}}, x_{n+\frac{1}{3}}\}$, yields the following three discrete schemes,

$$\begin{aligned} y_{n+\frac{1}{3}} - y_n &= h(\tfrac{1}{8} f_n + \tfrac{1}{72} f_{n+1} + \tfrac{19}{72} f_{n+\frac{1}{3}} - \tfrac{5}{72} f_{n+\frac{2}{3}}) \\ y_{n+\frac{2}{3}} - y_n &= h(\tfrac{1}{9} f_n + \tfrac{4}{9} f_{n+\frac{1}{3}} + \tfrac{1}{9} f_{n+\frac{2}{3}}) \\ y_{n+1} - y_n &= h(\tfrac{1}{8} f_n + \tfrac{1}{8} f_{n+1} + \tfrac{3}{8} f_{n+\frac{1}{3}} + \tfrac{3}{8} f_{n+\frac{2}{3}}) \end{aligned} \qquad (15)$$

This is the proposed discrete one-step two-off point continuous hybrid block collocation method (CHBCM) described by equation (14).

5.0 ANALYSIS

In this section, we conduct the analysis of the newly developed one-step two-off-point schemes given in equation (15) in terms of zero stability, consistency, convergence and region of absolute stability. In order to do so, we convert the method to its matrix form as seen below,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_{n+\frac{1}{3}} \\ y_{n+\frac{2}{3}} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_n \\ y_n \\ y_n \end{bmatrix} + h\begin{bmatrix} \frac{1}{8} & \frac{1}{72} \\ \frac{1}{9} & 0 \\ \frac{1}{8} & \frac{1}{8} \end{bmatrix}\begin{bmatrix} f_n \\ f_{n+1} \end{bmatrix} + h\begin{bmatrix} \frac{19}{72} & \frac{-5}{72} \\ \frac{4}{9} & \frac{1}{9} \\ \frac{3}{8} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} f_{n+\frac{1}{3}} \\ f_{n+\frac{2}{3}} \end{bmatrix} \quad (16)$$

5.1 Zero Stability. According to Fatunla (1992) a Continuous Hybrid Block Collocation Method (CHBCM) is zero stable, if the roots $R_j,\ j = 1(1)k$ of the characteristic polynomial $\rho(R)$ given as $\rho(R) = \det[\sum_{j=0}^{k} RA^i - A^1] = 0$ satisfies the condition $|R_j| \le 1$ (called the root contrition) and all roots $|R_j| = 1$ are of multiplicity of at most 2. That is if the roots of the characteristic polynomial $\rho(R)$ all have modulus less than or equal to 1 ($r \le 1$) and the roots of modulus 1 are of multiplicity1, we say that the *root condition* is satisfied. Thus to test the zero stability of the new method, it suffices to test the root condition. A linear multistep method is zero-stable if and only if the root condition is satisfied (Suli and Mayers, 2003). From equation (16), we have

$$A^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } A^1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So that the characteristic polynomial $\rho(R)$ is given as

$$\rho(R) = \det\left(R\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right) = 0$$

$$\Rightarrow R^2(R-1) = 0 \quad (17)$$

Solving for the roots R_i, of equation (17) we get $R_1 = 0$, $R_2 = 0$ and $R_3 = 1$. Since the roots $R_j,\ j = 1,2,3$ of the characteristic polynomial satisfies $|R_j| \le 1,\ j = 1,2,3$, the block method is *zero stable*.

5.2 Order and Error Constants. According to Adesanya, Udoh and Ajileye (2013) a hybrid block method is consistent if it is of order of accuracy $p \ge 1$. To evaluate the order and error constant, we expand the methods in their respective Taylor's Series. The coefficients obtained from the expansion shows that $C_0 = C_1 = C_2 = C_3 = C_4 = 0$ and $C_5 \ne 0$ for all the three methods in the block, showing that it is of the order of accuracy $[4, 4, 4]^T$. Evaluating $C_5 \ne 0$ for all the three methods, we get the error constants $\left[-\frac{19}{174960}, -\frac{1}{21870}, -\frac{1}{6480}\right]^T$. This confirms the assertion of Akinfenwa, Jator, and Yao (2011), Ndam, Habu, and Segun, (2014) and Operea (2014) that hybrid methods possess remarkably small error constants. This shows that the block method is of order $p > 1$, showing that the method is consistent.

5.3 Convergence. The convergence of this block method follows from the fact that it is consistent and zero stable. Ndanusa (2007) declared that "to prove that a scheme converges, it is sufficient to show that it is consistent as well as zero stable". This is quit in agreement with Butcher (2008) which states that a method is convergent if and only if it is zero stable and consistent.

5.4 Region Absolute Convergence. In order to see the region of absolute stability of the block method, we plot it for each of the methods in the scheme using the boundary lacus method.

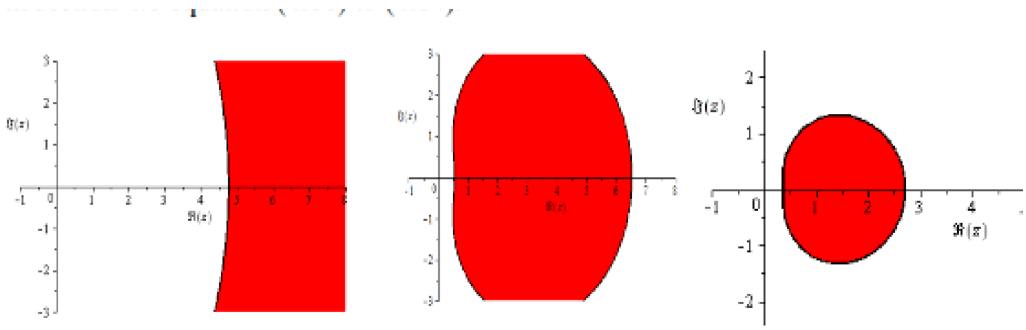

Figure 1: Regions of absolute stability

Remark: All the regions of absolute stability for the three methods are symmetrical about the axis. It can be seen from Figure 1 that the region of absolute stability of the block method is reasonably large enough to justify the stability of the method.

6.0 IMPLIMENTATION

In this section, the results obtained by implementing the new method on sample problems are presented in tabular form, showing the step size, h the computed results, the exact solution and the absolute error. The results were also compared with similar existing methods.

Example 1: Consider the initial value problem $y' = 8(x-y)+1$, $y(0)=1$, $0 \le x \le 1$, $h=0.1$, with exact solution $y(x) = x + 2e^{-8x}$.

Solution: By applying the new method, we get the solutions listed in Table 1below.

Table 1: Results for example 1

h	Computed results	Exact Solution	Absolute Error
0.1000	0.998657924	0.998657928	4.548E-09
0.2000	0.603793032	0.603793036	4.087E-09
0.3000	0.481435904	0.481435907	2.754E-09
0.4000	0.481524406	0.481524408	1.650E-09
0.5000	0.536631277	0.536631278	9.268E-10
0.6000	0.616459494	0.616459494	4.998E-10
0.7000	0.707395727	0.707395727	2.620E-10
0.8000	0.803323114	0.803323115	1.345E-10
0.9000	0.901493172	0.901493172	6.800E-11
1.0000	1.000670925	1.000670925	3.395E-11

Remark: It can be seen that the method produced exact solution up to eight decimal places. It can be noted that the errors in the methods reduces sharply as h increases, implying the convergence of the method.

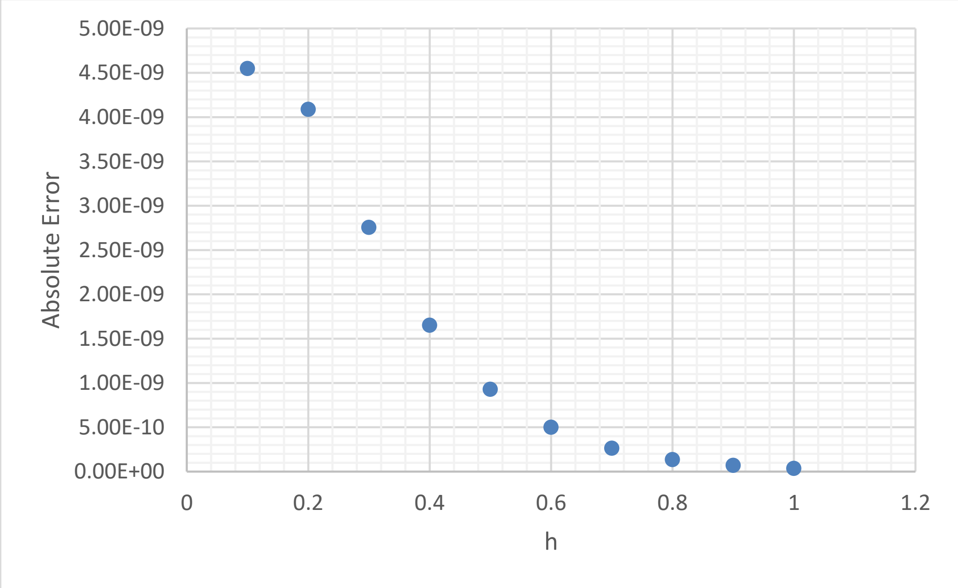

Figure 2: Absolute error

In order to compare the method with existing method, we tabulate the absolute errors of the method side by side with the method developed by Odejide and Adeniran (2012) abbreviated as O&A (2012) in Table 2 below.

Table 2. Comparison of absolute error for example 1

h	O&A (2012)	CHBCM
0.1000	8.071E-14	4.548E-09
0.2000	3.865E-14	4.087E-09
0.3000	1.444E-14	2.754E-09
0.4000	7.305E-14	1.650E-09
0.5000	3.864E-14	9.268E-10
0.6000	7.550E-14	4.998E-10
0.7000	2.343E-14	2.620E-10
0.8000	2.709E-14	1.345E-10
0.9000	4.574E-14	6.800E-11
1.0000	3.952E-14	3.395E-11

From Table 2, it can be seen that Odejide and Adeniran (2012) of order p = 7 have lesser absolute error. However, if we consider the order of the new methods, it proved more effective and since the errors reduces as h increases the method converge faster than Odejide and Adeniran (2012).

Example 2: Consider the initial value problem $y' = xy$, $y(0) = 1$, $0 \le x \le 1$, $h = 0.1$, with exact solution given by $y(x) = e^{\frac{1}{2}x^2}$.

Solution: By applying the new method, we get the solutions listed in Table 3 below:

Table 3: Results for example 2

H	Computed results	Exact Solution	Absolute Error
0.1000	1.005012522	1.005012521	1.165E-09
0.2000	1.020201345	1.020201340	4.778E-09
0.3000	1.046027871	1.046027860	1.120E-08
0.4000	1.083287089	1.083287068	2.111E-08
0.5000	1.133148489	1.133148453	3.551E-08
0.6000	1.197217419	1.197217363	5.592E-08
0.7000	1.277621398	1.277621313	8.455E-08

0.8000	1.377127889	1.377127764	1.245E-07
0.9000	1.499302680	1.499302500	1.803E-07
1.0000	1.648721529	1.648721271	2.584E-07

Remark: From the results in Table 3,it can be seen that the method performed well, even though the errors seems to grow (see Figure 3) as h increases indicating slower convergence.

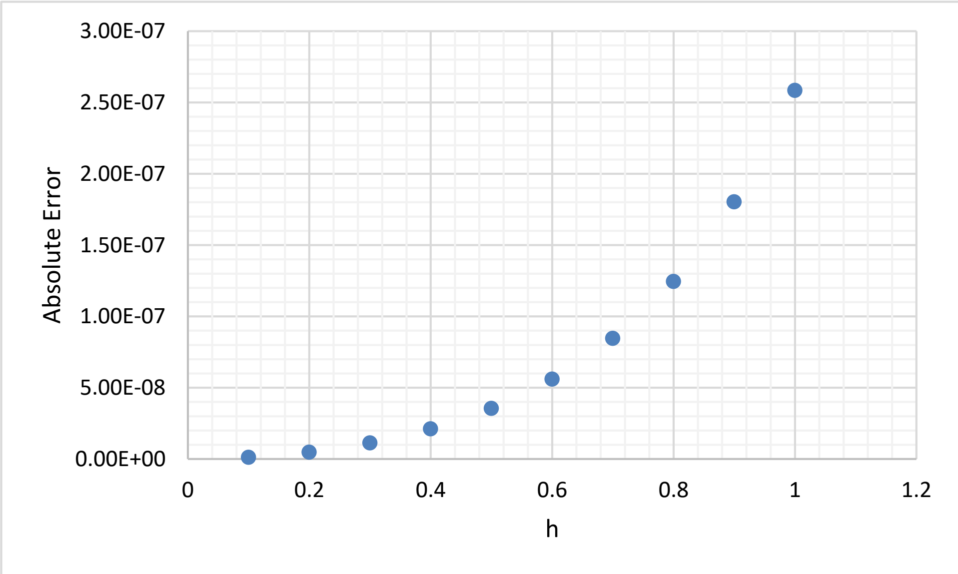

Figure 3: Absolute Error

In order to compare the method with existing method, we tabulate the absolute errors of the method side by side with Odekunle, Adesanya and Sunday (2012) abbreviated as AOS (2012) in Table 4 below.

Table 4: Comparison of absolute error for example 2

h	OAS (2012)	CHBCM
0.1000	5.240E-07	1.165E-09
0.2000	1.691E-07	4.778E-09
0.3000	8.724E-06	1.120E-08
0.4000	3.010E-06	2.111E-08
0.5000	1.747E-06	3.551E-08
0.6000	4.171E-06	5.592E-08
0.7000	9.647E-06	8.455E-08
0.8000	6.799E-06	1.245E-07
0.9000	1.291E-05	1.803E-07
1.0000	2.658E-05	2.584E-07

From the above table 4, we can see that the new methods have lesser error compared to the order $p = 4$ method developed by Odekunle, Adesanya and Sunday (2012). More so, if we consider the issue of order $p = 3$, the new method proved effective. Fortunately, we also observed that the error reduced as h increases showing that the two methods will converge faster than Odekunle, Adesanya and Sunday (2012).

Example 3: Consider the initial value problem $y' = -y^2, y(0) = 1;\ 0 \le x \le 1, h = 0.1$, with exact solution given by $y(x) = \frac{1}{1+x}$.

Solution. The results obtained by applying the new method are as presented in Table 5.

Table 5: Results for example 3

H	Computed results	Exact Solution	Absolute Error
0.1000	0.909090909	0.909090909	1.269E-11
0.3000	0.769230769	0.769230769	1.990E-11
0.4000	0.714285714	0.714285714	2.002E-11
0.5000	0.666666667	0.666666667	1.931E-11
0.6000	0.625000000	0.625000000	1.823E-11
0.7000	0.588235294	0.588235294	1.701E-11
0.8000	0.555555556	0.555555556	1.578E-11
0.9000	0.526315789	0.526315789	1.461E-11
1.0000	0.500000000	0.500000000	1.350E-11

Remarks: From Table 4, it can be noted that the method produced exact solutions. It can also be seen that the error reduces as *h* increases though slowly. Figure 4 illustrates the nature of distribution of the errors in the two methods.

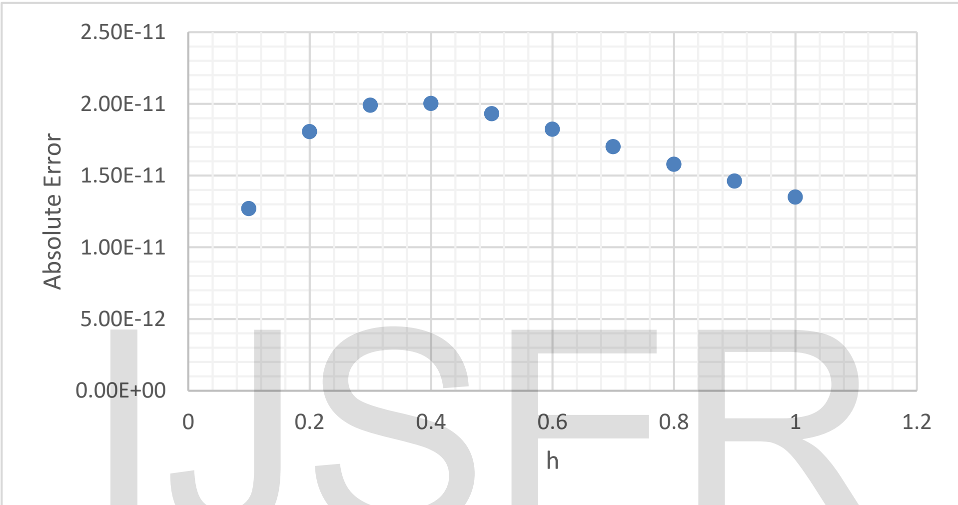

Figure 4: Absolute Error

In order to compare the method with existing method, we tabulate the absolute errors of the method side by side with Odejide and Adeniran (2012) shortened as O&A (2012) in Table 6.

Table 6. Comparison of absolute error for example 3

h	O&A (2012)	CHBCM (K = 1)
0.1000	2.918E-11	1.269E-11
0.2000	3.716E-11	1.806E-11
0.3000	3.937E-11	1.990E-11
0.4000	3.400E-11	2.002E-11
0.5000	2.949E-11	1.931E-11
0.6000	2.613E-11	1.823E-11
0.7000	2.315E-11	1.701E-11
0.8000	6.807E-06	1.578E-11
0.9000	8.317E-06	1.461E-11
1.0000	7.507E-06	1.350E-11

Comparing the error in the methods with order P = 7 hybrid method developed by Odejide and Adeniran (2012), it can be observed that the new methods performed better. Taking into cognizance, the fact that is order p = 3 the method performed excellently better. More so can see that the error reduces with increase *h* while the reverse is the case in Odejide and Adeniran (2012).

7.0 CONCLUSION

The purpose of this research was to develop and apply a new self-starting and implicit one step Continuous Hybrid Collocation Scheme for the integration of Initial Value Problems of the form in equation (1). This was successfully achieved by the assuming a power series solution, applying interpolation and collocation approach on the off-points of the closed interval $[0, 1]$, which resulted to a 5-equation in 5-unknowns. Solving and substituting the values resulted to the three discrete schemes presented in equation (15). The analysis shows that the scheme was zero stable, consistent and convergent. The block form of the method was implemented on sample initial value problems. The results shows that the scheme was effective in integrating initial value problems, since it produced exact solutions up to eight decimal places. In comparison to similar existing schemes, the method performed well. The method is hereby recommended.

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